

Вариант 11

МНК найти оценки параметров след. модели

$$y_t = \beta(1 + X_t) + \varepsilon_t = \beta + \beta X_t + \varepsilon_t$$

$$F = \sum_{t=1}^n (y_t - (\beta + \beta X_t))^2 \rightarrow \min$$

Необх. усл. экстремума:

$$\frac{\partial F}{\partial \beta} = -2 \sum_{t=1}^n (1 + X_t)(y_t - \beta - \beta X_t) = 0$$

$$\sum_{t=1}^n (y_t + X_t y_t - \beta - \beta X_t - \beta X_t - \beta X_t^2) = 0$$

$$\sum_{t=1}^n y_t + \sum_{t=1}^n X_t y_t = \beta n + \beta \sum_{t=1}^n X_t + \beta \sum_{t=1}^n X_t + \beta \sum_{t=1}^n X_t^2$$

$$\sum_{t=1}^n y_t + \sum_{t=1}^n X_t y_t = \beta \left(n + 2 \sum_{t=1}^n X_t + \sum_{t=1}^n X_t^2 \right)$$

$$\boxed{\beta = \frac{\sum_{t=1}^n y_t + \sum_{t=1}^n X_t y_t}{n + 2 \sum_{t=1}^n X_t + \sum_{t=1}^n X_t^2}}$$